32.13 A parallel flow heat exchanger is required to heat $30gpm$ of water from $50°F$ to $140°F$ using a heating medium which enters at $240°F$ and has a specific heat capacity of $0.7\frac{Btu}{lb_m \cdot °F}$ and a mass flow rate of $90,000\frac{lb_m}{hr}$. Selecting a counterflow heat exchanger has been proposed as an alternative. Neglecting losses, which of the following statements is true?

A. The parallel flow heat exchanger has a greater log mean temperature difference than the counterflow heat exchanger.

B. The counterflow heat exchanger has a greater log mean temperature difference than the parallel flow heat exchanger.

C. The counterflow heat exchanger and the parallel flow heat exchanger have the same log mean temperature difference.

D. It is not possible to determine whether the counterflow heat exchanger or the parallel flow heat exchanger has a greater log mean temperature difference.

Assuming 100% efficiency, the heat gained by the water is equal to the heat removed from the heating medium. Write expressions for both heat transfers and set them equal. Solve for the exit temperature of the heating medium. Use the sensible heating rule of thumb for water, noting the units work out to $\frac{Btu}{hr}$.

$$\dot{Q}_{water} = \dot{Q}_{htg}$$

$$500gpm\Delta T_{water} = \dot{m}c_p\Delta T_{htg}$$

$$(500)(30)(140-50)\frac{Btu}{hr} = \left(90,000\frac{lb_m}{hr}\right)\left(0.7\frac{Btu}{lb_m \cdot °F}\right)(240°F - T_2)$$

$$T_2 = 218.6°F$$

Calculate the Log Mean Temperature Difference for the default option of using a Parallel Flow heat exchanger. Draw the heat exchanger and label the temperatures.

$$Hot\,Fluid:\ 240°F \longrightarrow 218.6°F$$

$$Cold\,Fluid:\ 50°F \longrightarrow 140°F$$

Define one ***physical*** side of the heat exchanger as 'A' and the other side as 'B' and determine the respective temperature differences. Conveniently, the assignment of labels A and B turns out to be arbitrary. However, the *direction* of the flows is critical.

$$\Delta T_A = 240°F - 50°F = 190°F$$

$$\Delta T_B = 218.6°F - 140°F = 78.6°F$$

Use the formula below to calculate the log mean temperature difference.

$$LMTD_{parallel} = \frac{\Delta T_A - \Delta T_B}{ln\left(\frac{\Delta T_A}{\Delta T_B}\right)}$$

$$LMTD_{parallel} = \frac{190^\circ F - 78.6^\circ F}{ln\left(\frac{190^\circ F}{78.6^\circ F}\right)} = 126.2^\circ F$$

Calculate the log mean temperature difference for the alternate option of using a Counterflow heat exchanger. Draw the heat exchanger and label the temperatures.

$$Hot\,Fluid:\ 240^\circ F \longrightarrow 218.6^\circ F$$

$$Cold\,Fluid:\ 140^\circ F \longleftarrow 50^\circ F$$

Define one *physical* side of the heat exchanger as 'A' and the other side as 'B' and determine the respective temperature differences.

$$\Delta T_A = 240^\circ F - 140^\circ F = 100^\circ F$$

$$\Delta T_B = 218.6^\circ F - 50^\circ F = 168.6^\circ F$$

Use the formula below to calculate the log mean temperature difference.

$$LMTD_{counterflow} = \frac{\Delta T_A - \Delta T_B}{ln\left(\frac{\Delta T_A}{\Delta T_B}\right)}$$

$$LMTD_{counterflow} = \frac{100^\circ F - 168.6^\circ F}{ln\left(\frac{100^\circ F}{168.6^\circ F}\right)} = 131.3^\circ F$$

$$LMTD_{counterflow} > LMTD_{parallel}$$

Answer B